Internal Pair Emission from Aligned Nuclei

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The general correlation function describing the emission of positron-electron pairs from aligned nuclei is obtained from a plane-wave calculation. Results are presented and discussed for pure magnetic and pure electric transitions and for mixed transitions. It is assumed that the emitting state, although aligned, is unpolarized and that the polarization of neither member of the pair is observed. The dependence of internal pair emission on the nuclear alignment is compared with that of the competing gamma radiation. The behavior of the correlation function in the high-energy limit is also discussed.

I. INTRODUCTION

RECENTLY, a new method has been developed for determining the multipolarities of electromagnetic transitions by studies of the internal pairs with a magnetic pair spectrometer.[1] To obtain the full benefit from this method, it is necessary to have a more complete description of the emission of internal pairs from nuclei than has been given previously. Specifically, what is needed is a rather complete description of the emission of internal pairs from aligned nuclei; that is, emission of internal pairs from an excited state which has been formed by a nuclear reaction which establishes an axis of rotational symmetry. The purpose of this report is to present such a description.

The problem of internal pair emission was first treated in a general way for unoriented nuclei by Rose[2] who used the Born approximation to derive a complete description of the emission of internal pairs for the case when no quantization axis is established for the emitting nuclear state. This work shows clearly the dependence of internal pair emission upon the multipolarity of the transition. Later the region of validity of the Born approximation was delineated[3] and the approximation was shown to be rather good over a wide range of transition energies and nuclear charge. More specifically, the Born approximation is quite adequate for the purpose of determining multipolarities by study of the internal pairs emitted by light nuclei.[1,4]

The results of Rose were later extended by Goldring[5] to a discussion of the emission of internal pairs for the case of axially symmetric alignment of the nuclei. However, Goldring gave explicit results only for the special case of internal pair emission at right angles to the axis of alignment. Recently, Rose[6] has considered the emission of internal pairs from aligned nuclei; however, he restricted his discussion to a description of pure multipole emission after an average over one angle (describing the orientation of the plane defined by the positron-electron pair) had been performed. His results were identical to those given implicitly by Goldring, but displayed in a clearer way the physical significance of the various terms and the dependence on the quantities parametrizing the process.

In this report the full angular correlation function which describes internal pair emission from aligned nuclei is presented and its physical significance is discussed. Pure multipole transitions are considered in Sec. II, while mixed transitions are considered in Sec. III. The method used to calculate the angular correlation function is not discussed since it has been described fully by Rose[2,6] and Goldring.[5] The results given here have already been used to calculate the response of a magnetic pair spectrometer to internal pairs emitted by aligned nuclei.[1,7]

II. PURE MULTIPOLE EMISSION

The probability of a pair being emitted per quantum into solid angles $d\Omega_-$, $d\Omega_+$ and energy interval between W_+ and $W_+ + dW_+$ is defined as

$$F_L(\Theta,\delta,\theta,W_+)d\Omega_+ d\Omega_- dW_+\,,$$

where the subscripts $+,-$ refer to the positron and electron, respectively, and

$$d\Omega_+ d\Omega_- = \sin\Theta d\Theta\ \sin\theta d\theta d\delta d\phi\,.$$

The positron and electron energies are related by $W_+ + W_- = k$ where k is the energy of the transition in units of the electron rest mass; Θ is the angle between the positron and electron momenta, $\mathbf{p}_+$ and $\mathbf{p}_-$; θ is the angle between the z axis (axis of alignment) and the recoil direction, $\mathbf{q} = \mathbf{p}_+ + \mathbf{p}_-$; ϕ is the azimuthal angle of $\mathbf{q}$; and δ is the dihedral angle between the $(\mathbf{p}_+,\mathbf{p}_-)$ and $(\mathbf{q},\mathbf{z})$ planes. The correlation function $F_L(\Theta,\delta,\theta,W_+)$ also depends on the multipole order L of the transition and the nuclear parity change which collectively are referred to as the multipolarity. Note that $F_L(\Theta,\delta,\theta,W_+)$ is independent of ϕ since the z axis is an axis of rotational symmetry. The total pair formation coefficient for

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[4] D. H. Wilkinson, D. E. Alburger, E. K. Warburton, and R. E. Pixley, Phys. Rev. **129**, 1643 (1963).

[5] G. Goldring, Proc. Phys. Soc. (London) **A206**, 521 (1951).

[6] M. E. Rose, Phys. Rev. **131**, 1260 (1963).

[7] E. K. Warburton, D. E. Alburger, and D. H. Wilkinson, Phys. Rev. **132**, 776 (1963).

a particular multipolarity and transition energy is then[8]

$$\Gamma_L=\int F_L(\Theta,\delta,\theta,W_+)d\Omega_- d\Omega_+ dW_+. \qquad (1)$$

A straightforward but lengthy calculation based on the methods described by Rose[2,6] and Goldring[5] gives for both pure electric (EL) and pure magnetic (ML) transitions

$$8\pi^2F_L(\Theta,\delta,\theta,W_+)=\sum_\nu A_\nu(LL)[P_\nu(\cos\theta)\gamma_L{}^e(\Theta,W_+)+(-)^\sigma\cos2\delta\kappa_\nu(LL)P_\nu{}^{(2)}(\cos\theta)L_L(\Theta,W_+)]$$
$$+\sigma\frac{L}{L+1}\gamma_L{}^l(\Theta,W_+)\sum_\nu A_\nu(LL)\kappa_\nu{}^{(0)}(LL)P_\nu(\cos\theta)+2\sigma\left[\frac{L}{L+1}\right]^{1/2}\gamma_L{}^i(\Theta,W_+)$$
$$\times\cos\delta\sum_\nu A_\nu(LL)\kappa_\nu{}^{(1)}(LL)P_\nu{}^{(1)}(\cos\theta), \qquad (2)$$

where $\sigma=1$ for EL transitions and $\sigma=0$ for ML transitions, $P_\nu(\cos\theta)$ is a Legendre polynomial, $P_\nu{}^{(1)}(\cos\theta)$ and $P_\nu{}^{(2)}(\cos\theta)$ are associated Legendre functions of rank 1 and 2, respectively. The function $\kappa_\nu(LL)$, which is given by

$$\kappa_\nu(LL')=-\left[\frac{(\nu-2)!}{(\nu+2)!}\right]^{1/2}\frac{C(LL'\nu;11)}{C(LL'\nu;1,-1)}, \qquad (3)$$

where $L=L'$, is tabulated by Fagg and Hanna.[9] In Eq. (3), $C(LL'\nu;11)$ and $C(LL'\nu;1,-1)$ are vector-addition (Clebsch-Gordan) coefficients. The functions $\kappa_\nu{}^{(0)}(LL)$ and $\kappa_\nu{}^{(1)}(LL)$ are given by

$$\kappa_\nu{}^{(n)}(LL')=-\left[\frac{(\nu-n)!}{(\nu+n)!}\right]^{1/2}\frac{C(LL'\nu;n0)}{C(LL'\nu;1,-1)}, \qquad (4)$$

where $L=L'$.

The nuclear alignment is described by the $A_\nu(LL)$ which are the angular distribution coefficients for spin-one radiation and can be expressed in terms of statistical tensors α_ν by[6]

$$A_\nu(LL')=\alpha_\nu F_\nu(LL'J_fJ_i), \qquad (5)$$

where $L=L'$. The statistical tensors are normalized by $\alpha_0=1$, J_i and J_f refer to the angular momenta of the initial and final states, and $A_0(LL)=1$. The $F_\nu(LLJ_fJ_i)$ were first introduced by Biedenharn and Rose.[10] The most complete compilation of the functions $F_\nu(LL'J_fJ_i)$ is that given by Ferentz and Rosenzweig.[11] The $A_\nu(LL)$ can also be given in terms of the relative populations of the substates of the emitting level by

$$A_\nu(LL')=(-)^{L'-L}[(2L+1)(2L'+1)]^{1/2}\times C(LL'\nu;1,-1)\sum_{m_im}(-)^{m+1}$$
$$\times C(J_iLJ_f;m_im)C(J_iL'J_f;m_im)\times C(LL'\nu;m,-m)P(m_i), \qquad (6)$$

where

$$L=L' \quad\text{and}\quad \sum_{m_i}P(m_i)=1.$$

These two description are related by

$$\alpha_\nu=(2\nu+1)^{1/2}\sum_{m_i}C(J_i\nu J_i;m_i0)P(m_i). \qquad (7)$$

In Eqs. (6) and (7) m_i and m_f are the projections of J_i and J_f on the z axis, and $m=m_i-m_f$. It is assumed that there is no polarization of the initial state so that $P(m_i)=P(-m_i)$ which restricts ν to even values and ν runs from 0 to the smaller of $2L$ and $2J_i$. The $A_\nu(LL)\kappa_\nu{}^{(0)}(LL)$ occurring in the second sum of Eq. (2) are the angular distribution coefficients for spin-zero radiation.[12]

The correlation function for nonaligned nuclei is

$$F_L(\Theta,\delta,\theta,W_+)=\frac{1}{8\pi^2}\gamma_L(\Theta,W_+)$$
$$=\frac{1}{8\pi^2}\left[\gamma_L{}^e(\Theta,W_+)+\sigma\frac{L}{L+1}\gamma_L{}^l(\Theta,W_+)\right], \qquad (8)$$

since $P_\nu{}^{(n)}(\cos\theta)=0$ for $\nu<|n|$ and $A_\nu(LL)=0$ if $\nu\neq0$ for nonaligned nuclei. The $\gamma_L(\Theta,W_+)$ are given by Rose[2] for both EL and ML radiation.[13]

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[8] In the paper cited in Ref. 7, the notation used was θ for Θ, θ_q and ϕ_q for θ and ϕ, l for L, and $F_l(\theta,\delta,\theta_q,\phi_q)$ or $F_l(\theta,\delta,\theta_q)$ for $F_L(\Theta,\delta,\theta,W_+)$. In the paper cited in Ref. 6 the correlation function $N_\pi(\Theta,\theta,\delta,W_+)$, the number of pairs emitted per unit time in the range $dW_+\sin\Theta d\Theta\sin\theta d\theta d\delta$, is used instead of $F_L(\Theta,\delta,\theta,W_+)$, the relation between them is

$$F_L(\Theta,\delta,\theta,W_+)=\frac{1}{2\pi}N_\pi(\Theta,\theta,\delta,W_+)/N_\gamma,$$

where N_γ is the total number of photons emitted per unit time.

[9] L. W. Fagg and S. S. Hanna, Rev. Mod. Phys. **31**, 711 (1959).

[10] L. G. Biedenharn and M. E. Rose, Rev. Mod. Phys. **25**, 729 (1953).

[11] M. Ferentz and N. Rosenzweig, Argonne National Laboratory Report ANL-5324 (unpublished).

[12] In the paper cited in Ref. 10, $\kappa_\nu{}^{(0)}(LL)$ is referred to as $b_\nu(ll;\alpha)$, while in Ref. 9, $\kappa_\nu{}^{(0)}(LL')$ is referred to as $b_\nu(LL')$.

[13] In the papers cited in Refs. 2 and 5, $\gamma_L(\Theta,W_+)$ is referred to as $\gamma_l(\Theta)$. In the papers cited in Refs. 4 and 7, $\gamma_L(\Theta,W_+)$ and $L_L(\Theta,W_+)$ are referred to as $\gamma_l(\theta)$ and $L_l(\theta)$.

As noted by Rose,[6] Eq. (8) expresses $\gamma_L(\Theta,W_+)$ in terms of a contribution from the transverse potential, $\gamma_L{}^e(\Theta,W_+)$, and a contribution from the longitudinal potential, $\gamma_L{}^l(\Theta,W_+)$. Expressions for these two are given by Rose[6]; alternate expressions are

$$\gamma_L{}^e(\Theta,W_+)=(q/k)^{2L-2\sigma-2}\gamma_{M1}(\Theta,W_+), \qquad (9)$$

and

$$\gamma_L{}^l(\Theta,W_+)=2(q/k)^{2L-4}\times[(q/k)^2\gamma_{E1}(\Theta,W_+)-\gamma_{M1}(\Theta,W_+)]. \qquad (10)$$

In Eqs. (9) and (10), $\gamma_{E1}(\Theta,W_+)$ and $\gamma_{M1}(\Theta,W_+)$ are the $\gamma_L(\Theta,W_+)$ for electric dipole and magnetic dipole transitions, respectively. The other Θ-dependent quantities in Eq. (2) are

$$L_L(\Theta,W_+)=\frac{2\alpha}{\pi k^3}\frac{p_+p_-}{(k^2-q^2)^2}\left(\frac{q}{k}\right)^{2L-2\sigma-2}(\mathbf{p}_+\times\mathbf{p}_-)^2, \qquad (11)$$

and

$$\gamma_L{}^i(\Theta,W_+)=\frac{2\alpha}{\pi k^4}\frac{p_+{}^2p_-{}^2}{(k^2-q^2)}\left(\frac{q}{k}\right)^{2L-4}(W_--W_+)\sin\Theta, \qquad (12)$$

where α is the fine structure constant.

The intensity distribution of the linearly polarized gamma rays emitted in competiton with the internal pairs is given by [9,10]

$$8\pi^2W_L(\theta,\beta)=\sum_\nu A_\nu(LL)[P_\nu(\cos\theta)+(-)^\sigma\cos2\beta\kappa_\nu(LL)P_\nu{}^{(2)}(\cos\theta)], \qquad (13)$$

where θ is, as in the case of internal pair emission, the angle between the momentum transfer $\mathbf{q}$ and the z axis; β is the angle between the polarization vector and the normal to the $(\mathbf{z},\mathbf{q})$ plane; and $W_L(\theta,\beta)$, which is the probability of a gamma ray being emitted into the solid angle $\sin\theta d\theta d\beta d\phi$, has been normalized so that $\int W_L(\theta,\beta)\sin\theta d\theta d\beta d\phi=1$.

Now consider the dependence of the correlation function given in Eq. (2) upon the angular variables. First of all, it follows from Eqs. (4) and (8) that, as expected,

$$\int F_L(\Theta,\delta,\theta,W_+)\sin\theta d\theta d\delta d\phi=\gamma_L(\Theta,W_+). \qquad (14)$$

Integrating Eq. (2) over δ and ϕ alone gives

$$F_L(\Theta,\theta,W_+)=\tfrac{1}{2}\gamma_L{}^e(\Theta,W_+)\sum A_\nu(LL)P_\nu(\cos\theta)+\frac{\sigma}{2}\frac{L}{L+1}\gamma_L{}^l(\Theta,W_+)\sum_\nu A_\nu(LL)\times\kappa_\nu{}^{(0)}(LL)P_\nu(\cos\theta), \qquad (15)$$

where $F_L(\Theta,\theta,W_+)$ is defined by

$$F_L(\Theta,\theta,W_+)=\int F_L(\Theta,\delta,\theta,W_+)d\delta d\phi. \qquad (16)$$

Equation (15) is the result recently obtained by Rose[6] for the correlation function summed over the orientation about the direction of propagation $\mathbf{q}$ of the $(\mathbf{p}_+,\mathbf{p}_-)$ plane.

Integrating Eq. (13) over β and ϕ gives

$$W_L(\theta)=\tfrac{1}{2}\sum_\nu A_\nu(LL)P_\nu(\cos\theta), \qquad (17)$$

where $W_L(\theta)=\int W_L(\theta,\beta)d\beta d\phi$. Thus it is clear that for ML transitions the angular distribution of $\mathbf{q}$ given by $F_L(\Theta,\theta,W_+)$ is the same as for the competing gamma radiation; while for EL radiation there is an additional term in $F_L(\Theta,\theta,W_+)$, due to the longitudinal part, for which $\mathbf{q}$ has the same angular distribution as for spin-zero radiation. The properties of Eq. (15) have been discussed more fully by Rose.[6]

It is clear that in Eq. (2) the second part of the first sum, i.e., $(-)^\sigma\cos2\delta L_L(\Theta,W_+)\sum_\nu A_\nu(LL)\kappa_\nu(LL)P_\nu{}^{(2)}(\cos\theta)$, has a dependence on θ and δ identical to the dependence on θ and β of the gamma-ray linear polarization term given in Eq. (13). Thus δ is identified as the angle between a polarization vector and the normal to the $(\mathbf{q},\mathbf{z})$ plane so that the polarization vector is the normal to the $(\mathbf{p}_+,\mathbf{p}_-)$ plane. This polarization term arises because the positron-electron pair defines a plane and is not due to the spins of the electron and positron [these were summed over to obtain Eq. (2)]. Since $L_L(\Theta,W_+)$ is proportional to $(\mathbf{p}_+\times\mathbf{p}_-)^2$, the linear polarization term vanishes when the $(\mathbf{p}_+,\mathbf{p}_-)$ plane is not defined, as is expected.

The last term of Eq. (2) is the interference term between the transverse and longitudinal parts. Since it is proportional to $\cos\delta$ and involves a sum over $P_\nu{}^{(1)}(\cos\theta)$ it has properties analogous to the vector polarization term encountered in the angular distributions of particles with nonzero spin.[14] However, the analogy is not exact since the coefficient of $P_\nu{}^{(1)}(\cos\theta)$, namely $A_\nu(LL)\kappa_\nu{}^{(1)}(LL)$, cannot be equated with that expected for the polarization of particles with arbitrary spin. In fact, as can be inferred from inspection of Eqs. (3) and (4), the coefficient $A_\nu(LL)\kappa_\nu{}^{(1)}(LL)$ is just that expected for the interference term between a part which has an intensity distribution like spin-one radiation and a part which has an intensity distribution like spin-zero radiation.

It is instructive to consider the high-energy limit of Eq. (2). Consider first the case where the electron and positron are emitted in the same directions, i.e., $\Theta=0°$. Since $L_L(\Theta,W_+)$ and $\gamma_L{}^i(\Theta,W_+)$ are proportional to $\sin^2\Theta$ and $\sin\Theta$, respectively, these terms are zero for $\Theta=0°$. For $k\gg1$ the transverse contribution $\gamma_L{}^e(\Theta,W_+)$ has a sharp peak around $\Theta=0°$, which increases with k; while the longitudinal contribution $\gamma_L{}^l(\Theta,W_+)$ is not strongly dependent on Θ for any k. Thus as $k\to\infty$, $\gamma_L{}^l(0°,W_+)/\gamma_L{}^e(0°,W_+)\to0$ and $F(0°,\delta,\theta,W_+)_\infty$, the

[14] S. Devons and L. J. B. Goldfarb, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 42, p. 362. (See pp. 443–8.)

asymptotic value for $F_L(0°,\delta,\theta,W_+)$ as $k\rightarrow\infty$, is

$$F_L(0°,\delta,\theta,W_+)_\infty = \frac{1}{8\pi^2}\gamma_L{}^e(0°,W_+)\sum_\nu A_\nu(LL)P_\nu(\cos\theta), \quad (18)$$

so that for $\Theta=0°$ the asymptotic angular distribution of **q** with respect to the z axis is the same for internal pair emission and gamma radiation for both EL and ML transitions.

In order to give a more detailed description of the high-energy behavior of Eq. (2), it is convenient to define the angle Θ_0 such that $1-\cos\Theta_0=1/k$. As discussed by Goldring[5] and by Devons and Goldring,[15] $\gamma_L(\Theta,W_+)$ is nearly independent of L but strongly dependent on k for $\Theta\ll\Theta_0$ while for $\Theta\gg\Theta_0$ the opposite is true. Consider first the angular region $\Theta\gg\Theta_0$. For this region $L_L(\Theta,W_+)/\gamma_L{}^e(\Theta,W_+)$ and $\gamma_L{}^l(\Theta,W_+)/\gamma_L{}^e(\Theta,W_+)$ approach limiting values as $k\rightarrow\infty$ which are functions of W_+ in the first instance and of W_+ and Θ in the second instance. For the special case $W_+=W_-=k/2$ (important in the application to the magnetic pair spectrometer), $L_L(\Theta,W_+)_\infty=\gamma_L{}^e(\Theta,W_+)_\infty$ for $\Theta>\Theta_0$, and in this case

$$F_{ML}(\Theta,\delta,\theta,W_+)_\infty=\gamma_L{}^e(\Theta,W_+)\sum[A_\nu(LL)P_\nu(\cos\theta)+\cos2\delta\kappa_\nu(LL)P_\nu{}^{(2)}(\cos\theta)] \quad (\text{for } W_+=W_-=k/2,\ \Theta>\Theta_0), \quad (19)$$

so that for these special conditions the dependence of the asymptotic correlation function of internal pairs for ML transitions on θ and δ is the same as for gamma radiation [see Eq. (13)]. For all values of W_+ and Θ, $\gamma_L{}^i(\Theta,W_+)/\gamma_L{}^e(\Theta,W_+)$ approaches zero as $k\rightarrow\infty$ so that the contribution of this interference term is negligible at high energies; however, there are no values of W_+ for which $\gamma_L{}^l(\Theta,W_+)/\gamma_L{}^e(\Theta,W_+)$ approaches zero as $k\rightarrow\infty$ for $\Theta>\Theta_0$, so that for EL transitions the dependence on θ for pair emission differs from that for gamma radiation in the range $\Theta>\Theta_0$ even for $k\gg1$.

In the range $\Theta<\Theta_0$, $L_L(\Theta,W_+)$ approaches a value intermediate between $\gamma_L{}^e(\Theta,W_+)$ and 0, and the ratio $\gamma_L{}^l(\Theta,W_+)/\gamma_L{}^e(\Theta,W_+)$ approaches zero as $k\rightarrow\infty$. Thus for this range the asymptotic expression for EL transitions is the same as for ML transitions but in neither case is the dependence on θ and δ exactly the same as for linearly polarized gamma radiation.

The approximations used to derive Eq. (2) are not all valid for $k\gg1$. In particular, at very high energies there can be a significant contribution to the pair emission from the interior of the nucleus. However, the asymptotic values discussed above are approached quite rapidly so that the asymptotic behavior is important for transition energies encountered in practice.

III. MIXED TRANSITIONS

The results of the last section can be extended to mixed ML, $EL+1$ or EL, $ML+1$ transitions. The intensity distribution of gamma radiation in mixed transition is conventionally expressed as a function of the amplitude ratio x of $L+1$ to L radiation, i.e., $x=(J_i\|L+1\|J_f)/(J_i\|L\|J_f)$. The amplitude ratio x applies to internal pair emission as well as to gamma emission,[16] so that the correlation function for internal pair emission in a mixed transition can be expressed by

$$8\pi^2F(\Theta,\delta,\theta,W_+)=[F_L+(-)^\sigma 2xF_{LL'}+x^2F_{L'}]/(1+x^2), \quad (20)$$

where $L'=L+1$. In Eq. (20) F_L and $F_{L'}$ are the correlation functions, given by Eq. (2), for transitions with multipolarities L and $L+1$, respectively. The interference term $F_{LL'}(\Theta,\delta,\theta,W_+)$ can be evaluated using the methods of Rose[2,6] and Goldring[5] with the result

$$8\pi^2F_{LL'}(\Theta,\delta,\theta,W_+)=\sum_\nu A_\nu(LL')[P_\nu(\cos\theta)\gamma_{ML}(\Theta,W_+)-(-)^\sigma\cos2\delta\kappa_\nu(LL')P_\nu{}^{(2)}(\cos\theta)L_{ML}(\Theta,W_+)] + \frac{L'-\sigma}{[L'(L'+1)]^{1/2}}\gamma_{L'}{}^i(\Theta,W_+)\cos\delta\sum_\nu A_\nu(LL')\kappa_\nu{}^{(1)}(LL')P_\nu{}^{(1)}(\cos\theta), \quad (21)$$

where $L'=L+1$. In Eqs. (20) and (21) σ applies to the radiation of multipolarity L, i.e., $\sigma=0$ for a ML, $EL+1$ mixture and $\sigma=1$ for an EL, $ML+1$ mixture. The $\kappa_\nu(LL')$ are given by Eq. (3), and the $A_\nu(LL')$ are given by Eq. (5) or (6). The $\kappa_\nu{}^{(1)}(LL')$ are given by Eq. (4) with $n=1$ which reduces to

$$\kappa_\nu{}^{(1)}(LL')=-[L'(L'+1)]^{1/2}/\nu(\nu+1) \quad (22)$$

for $L'=L+1$. The functions

$$\gamma_{ML}(\Theta,W_+) \quad \text{and} \quad L_{ML}(\Theta,W_+)$$

are for ML radiation and are therefore given by Eqs. (9) and (11), respectively, with $\sigma=0$.

Since the vector addition coefficient $C(LL'\nu;m,-m)$ for $\nu=0$ is given by

$$C(LL'0;m,-m)=(-)^{L-m}\delta_{LL'}/(2L+1)^{1/2}, \quad (23)$$

it can be seen from Eq. (6) that $A_0(LL')$ and $A_0(LL')\times\kappa_0{}^{(1)}(LL')$ are both equal to zero for $L'=L+1$. Also the sum over m_i and m is equal to zero if all the $P(m_i)$

[15] S. Devons and G. Goldring, Proc. Phys. Soc. (London) **A67**, 413 (1954).

[16] The validity of this statement and related problems has been studied by J. M. Eisenberg and M. E. Rose, Bull. Am. Phys. Soc. 8, 315 (1963).

are equal. Thus for nonaligned nuclei, $F_{LL'}(\Theta,\delta,\theta,W_+)$ vanishes and there is no interference. For nonaligned nuclei, then, the total pair formation coefficient for a mixed transition is given by

$$\Gamma=(\Gamma_L+x^2\Gamma_{L'})/(1+x^2). \quad (24)$$

The integral of $F_{LL'}(\Theta,\delta,\theta,W_+)$ over ϕ, δ, and θ is zero for $L\neq L'$. Thus the interference term does not contribute to the total integrated intensity and Eq. (24) holds for aligned nuclei as well.

The intensity distribution of the linearly polarized gamma radiation for a mixed transition can be written[9,10]

$$8\pi^2W(\theta,\beta) =\frac{W_L(\theta,\beta)+(-)^\sigma 2xW_{LL'}(\theta,\beta)+x^2W_{L'}(\theta,\beta)}{1+x^2}, \quad (25)$$

where $L'=L+1$, and where $W_L(\theta,\beta)$ and $W_{L'}(\theta,\beta)$ can be obtained from Eq. (13). The interference term is given by

$$8\pi^2W_{LL'}(\theta,\beta)=\sum_\nu A_\nu(LL')[P_\nu(\cos\theta) -(-)^\sigma\cos 2\beta\kappa_\nu(LL')P_\nu^{(2)}(\cos\theta)]. \quad (26)$$

Integrating Eq. (26) over ϕ and β gives the intensity distribution of the interference term observed for polarization insensitive detectors

$$W_{LL'}(\theta)=\tfrac{1}{2}\sum A_\nu(LL')P_\nu(\cos\theta), \quad (27)$$

so that

$$W(\theta)=\tfrac{1}{2}\sum_\nu A_\nu P_\nu(\cos\theta), \quad (28)$$

where

$$A_\nu=\frac{A_\nu(LL)+(-)^\sigma 2xA_\nu(LL')+x^2A_\nu(L'L')}{1+x^2}. \quad (29)$$

It is expected that the dependence on $\mathbf{q}$ of the intensity distribution of internal pairs for a mixed transition will be the same as for unpolarized gamma radiation for $\Theta=0$ and $k\to\infty$ as is the case for pure transitions. Using Eqs. (9) and (11) and the limit $(q/k)^2\to 1$ as $k\to\infty$ for $\Theta=0$, the high-energy limit of Eq. (20) for $\Theta=0$ is found to be

$$8\pi^2F(0^0,\delta,\theta,W_+)_\infty=\gamma_L{}^e(0^0,W_+)\sum_\nu A_\nu P_\nu(\cos\theta), \quad (30)$$

where A_ν is given by Eq. (29). Thus, using Eq. (28) it is seen that

$$F(0^0,\delta,\theta,W_+)_\infty=\frac{1}{4\pi^2}\gamma_L{}^e(0^0,W_+)W(\theta) \quad (31)$$

and the dependence of $F(0^0,\delta,\theta,W_+)_\infty$ on $\mathbf{q}$ is the same as for unpolarized gamma radiation as expected.

The behavior of $F(\Theta,\theta,W_+)$, the integral of $F(\Theta,\delta,\theta,W_+)$ over ϕ and δ, and the dependence on Θ of the high-energy limit will not be discussed since they can be inferred easily from the results of the last section.

A special case of experimental interest is the form of the interference function $F_{LL'}(\Theta,\delta,\theta,W_+)$ for $\delta=\theta=\pi/2$.[5] From Eq. (21), this is

$$8\pi^2F_{LL'}\left(\Theta,\frac{\pi}{2},\frac{\pi}{2},W_+\right) =\sum_\nu A_\nu(LL')[P_\nu(0)\gamma_{ML}(\Theta,W_+)+(-)^\sigma\kappa_\nu(LL') \times P_\nu^{(2)}(0)L_{ML}(\Theta,W_+)]. \quad (32)$$

It is clear from Eq. (32) that the interference term does not vanish for these conditions nor can the integral of the interference term over W_+ vanish for arbitrary L and ν. This is contrary to a statement of Goldring.[5] The discrepancy is due to the fact that the expression given by Goldring for the interference function is missing an important term.[17] It is apparent then that the interference term must be considered in a rigorous analysis of experimental results for the angular correlations of internal pairs even for detection at right angles to the quantization axis. This is contrary to the usual practice in the past.[18]

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[17] Equation (6) of Ref. 5 should read (in the notation of that paper)

$$f_{l,l+1}{}^m=\frac{\alpha k}{32\pi^3}p_+p_-\,\mathrm{Re}[[1+W_+W_--(\mathbf{p}_+,\mathbf{p}_-)]\{\mathbf{q},(\mathbf{a}_{lM}{}^{m*}\mathbf{x}\mathbf{a}_{l+1E}{}^m)\} +\{v_{l+1}{}^m(W_--W_+)+(\mathbf{p}_--\mathbf{p}_+,\,\mathbf{a}_{l+1E}{}^m)\}(\mathbf{p}_+\mathbf{x}\mathbf{p}_-,\mathbf{a}_{lM}{}^{m*})]$$

where Re denotes the real part of the following expression.

[18] See, for example, Ref. 15.